Online Appendix of "Time Lotteries"

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A Local Risk Attitudes Towards Time Lotteries

Consider the case of infinite and discrete time ($T = \mathbb{N}$) and recall that according to EDU preferences are represented by

$$V(p) = \mathbb{E}_p\big[D(t)u(x)\big],$$

where $u : X \to \mathbb{R}_+$ is continuous and strictly increasing, and $D : \mathbb{N} \to (0, 1]$ is a strictly decreasing discount function.

We say that a discount function is *discretely convex* if D is a convex function when defined, that is, if for all $t_1, t_2 \in \mathbb{N}$ and $\alpha \in (0, 1)$,

$$\alpha D(t_1) + (1-\alpha) D(t_2) \geq D(\alpha t_1 + (1-\alpha) t_2)$$

whenever $\alpha t_1 + (1-\alpha) t_2 \in \mathbb{N}$.

The following proposition establishes the relationship between attitudes towards time lotteries and the convexity of the discount function:

Proposition 1. *Suppose $T = \mathbb{N}$. Under EDU, preferences are RSTL if and only if D is discretely convex. Moreover, they cannot be RATL.*

Proof. First, we show that preferences are RSTL (RATL) if and only if D is discretely convex (concave). (A discount function is discretely concave if $-D$ is discretely convex.) The value of $\delta_{(x,\bar{t})}$ is

$$V\left(\delta_{(x,\bar{t})}\right) = \sum\nolimits_{\tau \neq \bar{t}} D(\tau)\, u(c) + D\left(\bar{t}\right) u(c+x),$$

whereas the value of the time lottery $p = \langle p_x(t), t\rangle_{t\in\mathbb{N}}$ with $\sum_t p_x(t)\, t = \bar{t}$ is

$$V(p) = \sum\nolimits_t p_x(t) \left[\sum\nolimits_{\tau \neq t} D(\tau)\, u(c) + D(t)\, u(c+x)\right]$$

With algebraic manipulations give:

$$V(p) \geq V\left(\delta_{(x,\bar{t})}\right) \Leftrightarrow \left[\sum p_x(t) D(t) - D(\bar{t})\right] \left[u(c+x) - u(c)\right] \geq 0,$$

which, because u is strictly increasing, holds if and only if D is convex.

Next, we show that D cannot be discretely concave. Suppose D is discretely concave, so that

$$D(t) \leq D(1) + (t-1)\left[D(2) - D(1)\right].$$

Taking $t \geq \frac{2D(1)-D(2)}{D(1)-D(2)}$ and using the fact that D is strictly decreasing, we obtain $D(t) < 0$, which contradicts the fact that the discount function is positive. $\square$

As argued in the text, attitudes towards time lotteries do not rely on the curvature of the utility function u since all options involve the same payments, although in different periods. But is it plausible to assume that the discount function is convex? As we pointed out previously, virtually all discount functions used in economics are convex. Indeed, if preferences that are either time-consistent or present-biased, they must be discretely convex. Therefore, with standard discount functions, the discounted utility model leaves no degrees of freedom on the risk attitude towards time lotteries.

The second part of Proposition 1 states that discount functions cannot be discretely concave, implying that we cannot have RATL.

In light of Proposition 1, we ask whether discounted utility can satisfy a local version of RATL. We say that preferences are *locally risk averse towards time lotteries at time* t if a sure payment at t is preferred to a random payment occurring at either $t-1$ or at $t+1$ with equal probabilities, that is,

$$V\left(\delta_{(x,t)}\right) \geq V\left(\langle 0.5, (x, t-1); 0.5, (x, t+1)\rangle\right)$$

for all $x \in [w, b]$. Similarly, we say that preferences are locally risk seeking at t if the reverse inequalities hold.

Our next proposition shows that even this weaker version of RATL is inconsistent with EDU. Thus, even if we are willing to abandon (global) convexity, it would be of limited help.

Proposition 2. *Suppose $T = \mathbb{N}$. Under EDU, the set of periods in which preferences are locally RATL is finite.*

Proof. The sequence $\{D(t)\}$ is monotone and bounded. Thus, by the Monotone Convergence Theorem, it converges to some number, say $\bar{d} \geq 0$. We need to show that the sequence $\{D(t+1) + D(t-1) - 2D(t)\}$ has no negative limit points:

$$\liminf_{t \to \infty} \left(D(t+1) + D(t-1) - 2D(t)\right) \geq 0.$$

Suppose this is not true. Then there exists $\epsilon > 0$ and a subsequence $\{D(t_k)\}$ such that

$$D(t_k + 1) + D(t_k - 1) - 2D(t_k) \leq -\epsilon$$

for all t_k. However, because $D(t_k)$ converges to $\overline{d}$, it follows that $D(t_k + 1) + D(t_k - 1) - 2D(t_k)$ converges to zero. Thus, there exists $\overline{t}_k$ such that for all $t > \overline{t}_k$,

$$-\frac{\epsilon}{2} \leq D(t_k + 1) + D(t_k - 1) - 2D(t_k) \leq \frac{\epsilon}{2},$$

which contradicts the previous inequality. □

B Proofs of the Results in the Appendix

B.1 Proof of Proposition 3

We will proof first of all the following lemma.

Lemma 1. *Suppose T is an interval. The following are equivalent:*

1. *$\succeq$ satisfies Outcome Monotonicity, Impatience, Independence, Continuity, and Risk Stationarity;*

2. *$\succeq$ admits a GEDU representation (u, β, ϕ) where ϕ is a power function, i.e., there exists $\sigma \in \mathbb{R}$ such that*

$$\phi(x) = \begin{cases} x^{\sigma} & \text{if} \quad \sigma > 0 \\ \ln(x) & \text{if} \quad \sigma = 0 \\ -x^{\sigma} & \text{if} \quad \sigma < 0. \end{cases}$$

3. *$\succeq$ admits one of the following three representations:*

   (a) *(EDU) there exist $\beta \in (0,1)$, a strictly increasing $u : [w,b] \to \mathbb{R}_{++}$ such that $\succeq$ is represented by*

   $$V(p) = \mathbb{E}_p(\beta^t u(x));$$

   (b) *(Ln-EDU) there exist $\beta \in (0,1)$, a strictly increasing $u : [w,b] \to \mathbb{R}_{++}$ such that $\succeq$ is represented by*

   $$V(p) = \mathbb{E}_p(\ln(\beta^t u(x)));$$

   (c) *(Negative-EDU) there exist $\beta > 1$, a strictly increasing $u : [w,b] \to \mathbb{R}_{--}$ such that $\succeq$ is represented by*

   $$V(p) = \mathbb{E}_p(\beta^t u(x)).$$

Proof. Suppose $\succeq$ satisfies Outcome Monotonicity, Impatience, Independence, Continuity, and Risk Stationarity. By Theorem 1, it admits a GEDU representation u, β, ϕ. Define $U = \{s \in \mathbb{R} : s = \beta^t u(x) \text{ for some } x \in [w, b] \text{ and } t \in T\}$. Suppose, by means of contradiction, that ϕ does not admit the functional form described in (2). Then, there must exist a simple probability distribution f over U, with $f = \sum f_i u_i'$ for $u_i' \in U$, as well as $z' \in U$ and $a \in (0,1)$ such that $\sum f_i \phi(u_i') = \phi(z')$ but $\sum f_i \phi(a u_i') \neq \phi(a z')$ and $a u_i' \in U$, $a z' \in U$.

Define $\bar{a} = \sup\{\hat{a} \in (0,1) : \sum f(u_i') \phi(\hat{a} u_i) \neq \phi(\hat{a} z')\}$. Consider $a', a'' \in (0,1]$ such that $a' \geq \bar{a} > a'' \geq a$, $\sum f(u_i')\phi(a' u_i') = \phi(a' z')$, $\sum f(u_i')\phi(a'' u_i') \neq \phi(a'' z')$, and $\frac{a''}{a'} > \beta$. (Their existence is guaranteed by definition of $\bar{a}$, considering a' equal or arbitrarily close but above $\bar{a}$, and a'' equal or arbitrarily close but below $\bar{a}$.)

Now define $c = \frac{a''}{a'}$ a simple probability distribution r over U, defined by $r(x) = f(a' x)$, thus $r = \sum r(u_i) u_i$ where $u_i = a' u_i'$ for all i. Define also $z \in [w, b]$ such that $z = a' z'$. Note that by construction we have that $\sum r_i \phi(u_i) = \phi(z)$ but $\sum r_i \phi(c u_i) \neq \phi(cz)$. Moreover, we have $c u_i \in U$, $cz \in U$.

Given β and c, find $s \in \mathbb{R}$ such that $\beta^s = c$. Since $c = \frac{a''}{a'} > \beta$, note that we must have $s < 1$.

For all u_i in the support of r, consider $x_i \in [w, b]$ and $t_i \in T$ such that $u_i = \beta^{t_i} u(x_i)$ and $t_i = \min\{t \in T : u_i = \beta^t u(x) \text{ for some } x \in [w, b]\}$. The existence of x_i and t_i is guaranteed by the fact that $u_i \in U$. Notice also that, by construction, we have $t_i = 0$ if and only if $x_i > w$. (If $x_i > w$ then for t_i to be the minimum possible it must be equal to 0.) Similarly, given z, consider $y \in [w, b]$ and $t \in T$ such that $\beta^t u(y) = z$ and $t = \min\{t \in T : z = \beta^{t'} u(x) \text{ for some } x \in [w, b]\}$.

Notice also that for all i, we must have $t_i + s \in T$. To see why, suppose this is not the case. Since T is an interval, this means $t_i + s > \max\{t \in T\}$. Recall now that we must have that $c u_i \in U$, which means that there must exist some $x' \in [w, b]$ and $t' \in T$ such that $c u_i = \beta^{t'} u(x')$. Since $t_i + s > \max\{t \in T\}$, we must have $t' < t_i + s$. But now notice that we have $\beta^{t'} u(x') = c u_i = \beta^s \beta^{t_i} u(x_i) = \beta^{t+s} u(x_i)$. Since $t' < t_i + s$, then we must have $x' < x_i$. Thus, we must have $x_i > w$, which as we have seen above implies $t_i = 0$. But since $s < 1$ and $1 \in T$, then $t_i + s \in T$, a contradiction.

An identical argument proves that we must have $t + s \in T$.

Now construct $p \in \Delta$ by $p((x_i, t_i)) = r(u_i)$ for all i. Notice that we have

$$V(p) = \sum_i p((x_i, t_i))\phi(\beta^{t_i} u(x_i)) = \sum_i r(u_i)\phi(u_i) = \phi(z) = V(\delta_{(y,t)}).$$

Thus, we must have $p \sim \delta_{(y,t)}$. At the same time, notice that p_{+s} and $\delta_{(y,t+s)}$ are well defined, since $t_i + s \in T$ for all i and $t + s \in T$. Now notice that

$$V(p_{+s}) = \sum_i p((x_i, t_i))\phi(\beta^{t_i + s} u(x_i)) = \sum_i r(u_i)\phi(\beta^s u_i) = \sum_i r(u_i)\phi(c u_i).$$

By construction, this is different from $\phi(cz) = \phi(\beta^s \beta^t u(y)) = \phi(\beta^{s+t} u(y)) = V(\delta_{(y,t+s)}$. Thus, $p_{+s} \sim \delta_{(y,t+s)}$, contradicting Risk Stationarity.

(2) implies (3) Note that there are three cases in (2). First, $\phi(x) = x^\alpha$ for $\alpha > 0$. Then, define $\bar{\beta} = \beta^\alpha < 1$ and $\bar{u}(x) = u(x)^\alpha$, thus we have $\phi(\beta^t u(x)) = \bar{\beta}^t u(x)$. This corresponds to (a), which is EDU. The second case is $\phi(x) = ln(x)$: this leads immediately to (b), Ln-EDU. The third case is $\phi(x) = -x^{-\alpha}$ for $\alpha > 0$. Define $\bar{\beta} = \beta^{-\alpha} > 1$ and $\bar{u}(x) = -u(x)^{-\alpha}$, where $\bar{u}$ is negative and strictly increasing, thus $\phi(\beta^t u(x)) = \bar{\beta}^t u(x)$. This is (c), Negative-EDU.

(3) implies (1) Follows trivially from standard arguments. □

We now turn to prove the main proposition.

By the assumptions and Lemma 1, $\succeq$ admits one of three representations: EDU, Ln-EDU, or Negative-EDU. Consider a time lottery p_x with prize x and denote $\bar{t} = \sum_\tau p_x((x,\tau))$.

Suppose $\succeq$ admits an EDU representation β, u. Then:

$$\begin{aligned} p_x &\succ \delta_{(x,\bar{t})} \quad \Leftrightarrow \\ \sum p_x((x,t))u(x)\beta^t &> \beta^{\bar{t}} u(x) \quad \Leftrightarrow \\ \sum p_x((x,t))\beta^t &> \beta^{\bar{t}}. \end{aligned}$$

where the latter is true because β^x is a convex function. Thus, if $\succeq$ admits an EDU representation, it must be RSTL and not RNTL.

Suppose $\succeq$ admits a Ln-EDU representation β, u. Then

$$\begin{aligned} p_x &\sim \delta_{(x,\bar{t})} \quad \Leftrightarrow \\ \sum p_x((x,t)) \ln\left(u(x)\beta^t\right) &= \ln\left(\beta^{\bar{t}} u(x)\right) \quad \Leftrightarrow \\ \ln(u(x)) + \sum p_x((x,t)) t \ln(\beta) &= \ln(u(x)) + \bar{t}\ln(\beta) \quad \Leftrightarrow \\ \sum p_x((x,t)) t \ln(\beta) &= \bar{t}\ln(\beta). \end{aligned}$$

Thus, if $\succeq$ admits an Ln-EDU representation, it must be RNTL.

Finally, suppose $\succeq$ admits an Negative-EDU representation β, u. Then:

$$\begin{aligned} p_x &\prec \delta_{(x,\bar{t})} \quad \Leftrightarrow \\ \sum p_x((x,t))u(x)\beta^t &< \beta^{\bar{t}} u(x) \quad \Leftrightarrow \\ \sum p_x((x,t))\beta^t &> \beta^{\bar{t}}. \end{aligned}$$

where the last passage has reversed signs since $u(x) < 0$, and the last part is true since β^x is a convex function. Thus, if $\succeq$ admits a Negative-EDU representation, then it must be RATL and not RNTL.

Since the three conditions, RSTL and not RNTL, RNTL, and RATL and not RNTL, are mutually exclusive, then each of the statements above must be an if and only if, proving the proposition. ■

B.2 Proof of Theorem 3

To prove that (1) implies (2), construct a preference relation $\succeq'$ on $\mathcal{X}$ by $x \succeq' y$ if and only if $\delta_x \succeq \delta_y$. Notice that $\succeq'$ satisfies weak ordering, monotonicity, ultimate continuity, initial trade-off independence, and stationarity of Bleichrodt et al. (2008). Notice also that, by Monotonicity, $\succeq'$ also satisfies the condition called 'sensitivity to the first period' in that paper. Then, by Theorem 2 as well as Observation 3 in that paper, we obtain that there exist a continuous $u : [w, b] \to \mathbb{R}$ and $\beta \in (0, 1)$ such that $\succeq'$ is represented by

$$U(x) := \sum_{t=0}^{\infty} \beta^t u(x_t).$$

where β is unique and u is unique up to a positive affine trasformation. Notice also that, by Monotonicity, u must be strictly increasing. By Independence and Archimedean Continuity (see Kreps 1988, Th. 5.15), there exists $U' : \mathcal{X} \to \mathbb{R}$ such that $\succeq$ is represented by

$$V'(p) := \mathbb{E}_p(U').$$

Notice also that U' must represent $\succeq'$. Thus, since both U and U' represent the same preferences, there must exist a strictly increasing $\phi : U(\mathcal{X}) \to \mathbb{R}$ such that $U'(x) = \phi(U(x))$. (This follows a standard argument, identical to the one used in the proof of Theorem 1 above. Since ϕ is strictly increasing, then we must have that $\succeq$ are represented also by $V(p) = \phi^{-1}V'(p)$, as sought. Since U' is unique up to a positive affine transformation, and so is u, then ϕ must be as well. The proof that (2) implies (1) follows standard arguments or from noting that Initial Trade-off independence, Streams Stationarity and Ultimate Continuity are equivalent to the corresponding properties in Bleichrodt et al. (2008). ■

B.3 Proof of Theorem 4

Suppose the individual consumes c in every period, and gets an additional prize in the time period specified by the lottery he faces. Let $r_x = 0.5\delta_{(x,t_1)} + 0.5\delta_{(x,t_3)}$ (so that $t = \frac{t_1+t_3}{2}$). We have

$$V(\delta_{(x,t)}) = \phi(D(t)u(x+c) + u(c)\textstyle\sum_{t'\neq 1} D(t'))$$

and

$$\begin{aligned} V(r_x) \quad &= \quad \pi(0.5)\phi(D(t_1)u(x+c) + u(c)\textstyle\sum_{t'\neq 0} D(t')) + \\ &\qquad (1-\pi(0.5))\phi(D(t_3)u(x+c) + u(c)\textstyle\sum_{t'\neq t_3} D(t')) \end{aligned}$$

So $\overline{\pi(0.5)}$ is such that $V(\delta_{(x,t)}) = V(r_x)$, or

$$\overline{\pi(0.5)} = \frac{\phi(D(t_2)u(x+c) + u(c)\sum_{t'\neq t} D(t')) - \phi(D(t_3)u(x+c) + u(c)\sum_{t'\neq t_3} D(t'))}{\phi(D(t_1)u(x+c) + u(c)\sum_{t'\neq t_1} D(t')) - \phi(D(t_3)u(x+c) + u(c)\sum_{t'\neq t_3} D(t'))} \in (0,1)$$

Suppose first that

$$D(t_1)(u(w+c)-u(c)) < D(t_2)(u(x+c)-u(c))$$

Then there is $x' < x$ such that $D(t_1)u(x'+c)+u(c)\sum_{t'\neq t_1} D(t') = D(t_2)u(x+c)+u(c)\sum_{t'\neq t_2} D(t')$. Let $p = 0.5\delta_{(x,t_1)} + 0.5\delta_{(x',t_2)}$ and $q = 0.5\delta_{(x',t_1)} + 0.5\delta_{(x,t_2)}$ We have

$$\begin{aligned} V(p) &= \pi(0.5)\phi(D(t_1)u(x+c)+u(c)\sum\nolimits_{t'\neq 0} D(t')) + \\ &\quad (1-\pi(0.5))\phi(D(t_2)u(x'+c)+u(c)\sum\nolimits_{t'\neq t_2} D(t')) \end{aligned}$$

and

$$V(q) = \phi(D(t_2)u(x+c)+u(c)\sum\nolimits_{t'\neq t_2} D(t'))$$

So $\widehat{\pi(0.5)}$ is such that $V(p)=V(q)$, or

$$\widehat{\pi(0.5)} = \frac{\phi(D(t_2)u(x+c)+u(c)\sum_{t'\neq t_2} D(t')) - \phi(D(t_2)u(x'+c)+u(c)\sum_{t'\neq t_2} D(t'))}{\phi(D(t_1)u(x+c)+u(c)\sum_{t'\neq t_1} D(t')) - \phi(D(t_2)u(x'+c)+u(c)\sum_{t'\neq t_2} D(t'))} \in (0,1)$$

We will be done if we show that

$$\phi\left[D(t_2)u(x'+c)+u(c)\sum\nolimits_{t'\neq t_2} D(t')\right] \leq \phi\left[D(t_3)u(x+c)+u(c)\sum\nolimits_{t'\neq t_3} D(t')\right],$$

since this implies that $\widehat{\pi(0.5)} \geq \overline{\pi(0.5)}$, which is what we want.

Recall that $D(t_1)u(x'+c) = D(t_2)u(x+c)+u(c)\sum_{t'\neq t_2} D(t') - u(c)\sum_{t'\neq t_1} D(t')$. We therefore need to show that

$$\begin{aligned} & D(t_2)u(x'+c)+u(c)\sum\nolimits_{t'\neq t_2} D(t') \qquad \text{(A1)} \\ =\ & \frac{D(t_2)}{D(t_1)}\left[D(t_2)u(x+c)+u(c)\sum\nolimits_{t'\neq t_2} D(t') - u(c)\sum\nolimits_{t'\neq t_1} D(t')\right] + u(c)\sum\nolimits_{t'\neq t_2} D(t') \\ =\ & \frac{D(t_2)^2}{D(t_1)}u(x+c) + u(c)\left[\frac{D(t_2)}{D(t_1)}\left(\sum\nolimits_{t'\neq t_2} D(t') - \sum\nolimits_{t'\neq t_1} D(t')\right) + \sum\nolimits_{t'\neq t_2} D(t')\right] \\ \leq\ & D(t_3)u(x+c)+u(c)\sum\nolimits_{t'\neq t_3} D(t') \end{aligned}$$

First let's look at the coefficient that multiple $u(c)$. Observe that

$$\begin{aligned} & \frac{D(t_2)}{D(t_1)}\left(\sum\nolimits_{t'\neq t_2} D(t') - \sum\nolimits_{t'\neq t_1} D(t')\right) + \sum\nolimits_{t'\neq t} D(t') - \sum\nolimits_{t'\neq t_3} D(t') \\ =\ & \frac{D(t_2)}{D(t_1)}[D(t_1)-D(t_2)] + D(t_3) - D(t_2) \\ =\ & D(t_2) - \frac{D(t_2)^2}{D(t_1)} + D(t_3) - D(t_2) \\ =\ & D(t_3) - \frac{D(t)^2}{D(t_1)} \end{aligned}$$

Now let's look at the coefficient that multiple $u(x+c)$. It is $\frac{D(t_2)^2}{D(t_1)}$ on the LHS of (A1) and $D(t_3)$ on the RHS of (A1). But $c > 0$ and therefore

$$\begin{aligned}
& D(t_2)u(x'+c) + u(c)\sum\nolimits_{t'\neq t_2} D(t') - D(t_3)u(x+c) + u(c)\sum\nolimits_{t'\neq t_3} D(t') \\
= \ & [u(x+c) - u(c)]\left[\frac{D(t_2)^2}{D(t_1)} - D(t_3)\right] \leq 0
\end{aligned}$$

which is negative since, by diminishing impatience, $\frac{D(t_2)}{D(t_3)} \leq \frac{D(t_1)}{D(t_2)}$, or $\frac{D(t_2)^2}{D(t_1)} - D(t_3) \leq 0$.

Suppose instead that

$$D(t_3)(u(b) - u(c)) > D(t_2)(u(x+c) - u(c)).$$

Then there is $x' > x$ such that $D(t_3)u(x'+c) + u(c)\sum_{t'\neq t_3} D(t') = D(t_2)u(x+c) + u(c)\sum_{t'\neq t_2} D(t')$. Let $p = 0.5\delta_{(x',t_2)} + 0.5\delta_{(x,t_3)}$ and $q = 0.5\delta_{(x,t_2)} + 0.5\delta_{(x',t_3)}$ We have

$$\begin{aligned}
V(p) \ = \ & \pi(0.5)\phi(D(t_2)u(x'+c) + u(c)\sum\nolimits_{t'\neq t_2} D(t')) + \\
& (1 - \pi(0.5))\phi(D(t_3)u(x+c) + u(c)\sum\nolimits_{t'\neq t_3} D(t'))
\end{aligned}$$

and

$$V(q) = \phi(D(t_2)u(x+c) + u(c)\sum\nolimits_{t'\neq t_2} D(t'))$$

So $\widehat{\pi(0.5)}$ is such that $V(p) = V(q)$, or

$$\widehat{\pi(0.5)} = \frac{\phi(D(t_2)u(x+c) + u(c)\sum_{t'\neq t_2} D(t')) - \phi(D(t_3)u(x+c) + u(c)\sum_{t'\neq t_3} D(t'))}{\phi(D(t_2)u(x'+c) + u(c)\sum_{t'\neq t_2} D(t')) - \phi(D(t_3)u(x+c) + u(c)\sum_{t'\neq t_3} D(t'))} \in (0,1)$$

We thus need to show that $D(t_1)u(x+c) + u(c)\sum_{t'\neq t_1} D(t') \geq D(t_2)u(x'+c) + u(c)\sum_{t'\neq t_2} D(t')$, since this implies that $\widehat{\pi(0.5)} \geq \overline{\pi(0.5)}$, which is what we want.

Recall that $D(t_3)u(x'+c) + u(c)\sum_{t'\neq t_3} D(t') = D(t_2)u(x+c) + u(c)\sum_{t'\neq t_2} D(t')$, hence we need to show that

$$\begin{aligned}
& D(t_2)u(x'+c) + u(c)\sum\nolimits_{t'\neq t_2} D(t') && \text{(A2)} \\
= \ & \frac{D(t_2)}{D(t_3)}\left[D(t_2)u(x+c) + u(c)\sum\nolimits_{t'\neq t_2} D(t') - u(c)\sum\nolimits_{t'\neq t_3} D(t')\right] + u(c)\sum\nolimits_{t'\neq t_2} D(t') \\
= \ & \frac{D(t_2)^2}{D(t_3)}u(x+c) + u(c)\left[\frac{D(t_2)}{D(t_3)}\left[\sum\nolimits_{t'\neq t_2} D(t') - \sum\nolimits_{t'\neq t_3} D(t')\right] + \sum\nolimits_{t'\neq t_2} D(t')\right] \\
\leq \ & D(t_1)u(x+c) + u(c)\sum\nolimits_{t'\neq t_1} D(t')
\end{aligned}$$

Let's look at the coefficients of $u(c)$. Observe that

$$\begin{aligned}
& \frac{D(t_2)}{D(t_3)}\left[\sum\nolimits_{t'\neq t_2} D(t') - \sum\nolimits_{t'\neq t_3} D(t')\right] + \sum\nolimits_{t'\neq t_2} D(t') - \sum\nolimits_{t'\neq t_1} D(t') \\
= & \frac{D(t_2)}{D(t_3)}\left[D(t_3) - D(t_2)\right] + D(t_1) - D(t_2) \\
= & D(t_2) - \frac{D(t_2)^2}{D(t_3)} + D(t_1) - D(t_2) = D(t_1) - \frac{D(t_2)^2}{D(t_3)} \geq 0
\end{aligned}$$

Where, by diminishing impatience, $D(t_1) - \frac{D(t_2)^2}{D(t_3)} \geq 0$. The coefficients of $u(x+c)$ are $\frac{D(t_2)^2}{D(t_3)}$ on the LHS of (A2) and $D(t_1)$ and on the RHS of (A2). Therefore,

$$\begin{aligned}
& D(t_2)u(x'+c) + u(c)\sum\nolimits_{t'\neq t_2} D(t') - D(t_1)u(x+c) + u(c)\sum\nolimits_{t'\neq t_1} D(t') \\
= & \left[\frac{D(t_2)^2}{D(t_3)} - D(t_1)\right]\left[u(x+c) - u(c)\right] \leq 0
\end{aligned}$$

which is negative since by diminishing impatience, $D(t_1) - \frac{D(t_2)^2}{D(t_3)} \geq 0$. The proof is now completed. ■

B.4 Proof of Proposition 4

The proof will use a couple of Lemmas.

First notice that, because preferences are dynamically consistent, there is no loss in taking $t = 3$. To simplify the expressions, it is convenient to write $\lambda \equiv (c+x)/c > 1$ to denote the consumption with the prize as a proportion of consumption without it. Using the formula in the text, the utility of the safe lottery equals

$$V_0 = \left[(1-\beta)\, c\right]^{\frac{1}{1-\rho}} \cdot \left[1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}\right]^{\frac{1}{1-\rho}},$$

and the utility of the risky lottery is

$$V_0 = \left[(1-\beta)\, c\right]^{\frac{1}{1-\rho}} \left\{1 + \beta\left[\frac{\left(\lambda^{1-\rho} + \frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}} + \left(1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}.$$

Therefore, preferences are locally RSTL at t if and only if the following inequality holds:

$$\left\{1 + \beta\left[\frac{\left(\lambda^{1-\rho} + \frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}} + \left(1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}} > \left(1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}\right)^{\frac{1}{1-\rho}}. \tag{A3}$$

To simplify notation, let $f(x) \equiv x^{\frac{1-\alpha}{1-\rho}}$. In the proofs, we will repeatedly use the following result. The expected discounted payoff from the risky lottery exceeds the one from the safe lottery if and only if the intertemporal elasticity of substitution exceeds 1. Formally:

$$\frac{\lambda^{1-\rho} + \frac{\beta}{1-\beta} + 1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}}{2} \left\{ \begin{array}{c} > \\ < \end{array} \right\} 1 + \lambda^{1-\rho}\beta + \frac{\beta^2}{1-\beta} \iff \rho \left\{ \begin{array}{c} < \\ > \end{array} \right\} 1. \tag{A4}$$

We first verify that (A3) always holds when $\alpha \leq 1$.

Lemma 1. *Let $\alpha \leq 1$. Then, preferences are RSTL.*

Proof. There are three cases: (i) $\alpha \leq \rho \leq 1$, (ii) $\rho < \alpha \leq 1$, and (iii) $\alpha \leq 1 < \rho$.

Case i: $\alpha \leq \rho \leq 1$. Since $1 - \rho < 0$, inequality (A3) can be written as

$$\frac{\left(\lambda^{1-\rho} + \frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}} + \left(1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2} > \left(1 + \lambda^{1-\rho}\beta + \frac{\beta^2}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}.$$

Algebraic manipulations establish that the expected discounted payment of the risky lottery exceeds the one from the safe lottery. Because $\rho < 1$, inequality (A4) gives

$$\frac{\lambda^{1-\rho} + \frac{\beta}{1-\beta} + 1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}}{2} > 1 + \lambda^{1-\rho}\beta + \frac{\beta^2}{1-\beta}.$$

The result then follows from Jensen's inequality since $f(x)$ is increasing and convex when $\alpha, \rho \leq 1$.

Case ii: $\rho < \alpha \leq 1$. To simplify notation, perform the following change of variables: $\gamma \equiv \frac{1-\alpha}{1-\rho} \in (0, 1)$ where $\gamma > 0$ since both α and ρ are lower than 1, and $\gamma < 1$ since $\alpha > \rho$. We can rewrite inequality (A3) substituting α for γ as

$$\frac{\left(\lambda^{1-\rho} + \frac{\beta}{1-\beta}\right)^{\gamma} + \left(1 + \beta + \lambda^{1-\rho}\beta^2 + \frac{\beta^3}{1-\beta}\right)^{\gamma}}{2} > \left(1 + \lambda^{1-\rho}\beta + \frac{\beta^2}{1-\beta}\right)^{\gamma}.$$

Rearrange this condition as

$$\left(\frac{1}{\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}} + \beta}\right)^{\gamma} + \left(\frac{1}{1 + \lambda^{1-\rho}\beta + \frac{\beta^2}{1-\beta}} + \beta\right)^{\gamma} > 2.$$

It is straightforward to verify that the expression on the left ("LHS") is a convex function of γ. Recall that $\gamma \in (0, 1)$. Evaluating at $\gamma = 0$, we obtain

$$LHS|_{\gamma=0} = 2.$$

Since LHS is a convex function of γ, it suffices to show that its derivative wrt γ at zero is positive. We claim that this is true. To see this, notice that

$$\left.\frac{dLHS}{d\gamma}\right|_{\gamma=0} = \ln\left(\frac{\frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta}{\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta}\right), \tag{A5}$$

which, with some algebraic manipulations, can be shown to be strictly positive for any $\rho < 1$. Thus, $LHS > 2$ for all $\gamma \in (0,1]$, establishing RSTL.

Case iii: $\alpha \le 1 < \rho$. Inequality (A3) can be simplified as

$$\left[\frac{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2}+\frac{\left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}} < 1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}.$$

Since $\frac{1-\alpha}{1-\rho} < 0$, this holds if

$$\frac{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}+\left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2} > \left(1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}. \tag{A6}$$

Notice that $f(x) = x^{\frac{1-\alpha}{1-\rho}}$ is convex since

$$f''(x) = \left(\frac{1-\alpha}{1-\rho}\right)\left(\frac{1-\alpha}{1-\rho}-1\right)x^{\frac{1-\alpha}{1-\rho}-2} > 0,$$

where we used $\frac{1-\alpha}{1-\rho} < 0$ and $\frac{1-\alpha}{1-\rho} - 1 < 0$. Thus, by Jensen's inequality,

$$\frac{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}+\left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2} > \left(\frac{\lambda^{1-\rho}+\frac{\beta}{1-\beta}+1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}}{2}\right)^{\frac{1-\alpha}{1-\rho}}. \tag{A7}$$

From condition (A4), we have

$$\frac{\lambda^{1-\rho}+\frac{\beta}{1-\beta}+1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}}{2} < 1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}.$$

Raising to $\frac{1-\alpha}{1-\rho} < 0$, gives

$$\left(\frac{\lambda^{1-\rho}+\frac{\beta}{1-\beta}+1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}}{2}\right)^{\frac{1-\alpha}{1-\rho}} > \left(1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}.$$

Substituting in (A7), we obtain

$$\frac{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}+\left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2}>\left(1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}},$$

which is precisely the condition for RSTL (A6).

□

Lemma 2. *Let $\alpha \leq \rho$. Then, preferences are RSTL.*

Proof. By Lemma 1, the result is immediate when $\alpha \leq 1$. Therefore, let $\alpha > 1$ (which, by the statement of the lemma, requires $\rho > 1$).

Rearranging inequality (A3), we obtain the following condition for RSTL:

$$\frac{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}+\left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}}{2}<\left(1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}\right)^{\frac{1-\alpha}{1-\rho}}. \tag{A8}$$

Moreover, from condition (A4), we have

$$\frac{\lambda^{1-\rho}+\frac{\beta}{1-\beta}+1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}}{2}<1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}.$$

Notice that $f(x)$ is increasing when $\alpha, \rho \geq 1$ and it is concave when $\rho \geq \alpha$. Then, condition (A8) follows by Jensen's inequality. □

We are now ready to prove the main result:

Proof of Proposition 4. First, suppose $\rho < 1$. Let $\gamma \equiv -\frac{1-\alpha}{1-\rho} \in (0, +\infty)$ so we can rewrite inequality (A3) in terms of γ and ρ as

$$\frac{1}{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^{\gamma}}+\frac{1}{\left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^{\gamma}}<\frac{2}{\left(1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}\right)^{\gamma}},$$

which can be simplified as:

$$\left(\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta\right)^{\gamma}+\left(\frac{1}{\frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta}\right)^{\gamma}<2.$$

The first term in the expression on the left ("LHS") is convex and decreasing in γ, because the term inside the first brackets is smaller than 1:

$$\rho \leq 1 \implies \frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta \leq 1$$

The second term is convex and increasing in γ because the term inside the second brackets is greater than 1:

$$\rho \leq 1 \implies \frac{1}{\frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta} \geq 1.$$

Since the sum of convex functions is convex, it follows that LHS is a convex function of γ.

Evaluating γ at the extremes, we obtain:

$$LHS|_{\gamma=0} = \left(\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta\right)^0 + \left(\frac{1}{\frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta}\right)^0 = 2,$$

and

$$\lim_{\gamma\to\infty} LHS = +\infty > 2.$$

Moreover, we claim that the derivative of the LHS wrt γ at zero is negative. To see this, note that

$$\left.\frac{dLHS}{d\gamma}\right|_{\gamma=0} = \ln\left(\frac{\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta}{\frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta}\right),$$

which, following some algebraic manipulations, can be shown to be strictly negative.

Thus, there exists $\bar{\gamma} > 0$ such that $LHS > 2$ (RATL) if and only if $\gamma > \bar{\gamma}$. But, since $\gamma \equiv -\frac{1-\alpha}{1-\rho}$ (so that γ is strictly increasing in α), this establishes that there exists a finite $\bar{\alpha}_{\rho,\beta} > \max\{1,\rho\}$ such that we have RATL if $\alpha > \bar{\alpha}_{\rho,\beta}$ and RSTL if $\alpha < \bar{\alpha}_{\rho,\beta}$. This concludes the proof for $\rho < 1$.

Now suppose that $\alpha > \rho \geq 1$ (the result is trivial if $\alpha \leq \rho$ from Lemma 2). Let $\gamma \equiv \frac{1-\alpha}{1-\rho} \geq 1$. Then, we have RSTL if and only if

$$\frac{\left(\lambda^{1-\rho}+\frac{\beta}{1-\beta}\right)^\gamma + \left(1+\beta+\lambda^{1-\rho}\beta^2+\frac{\beta^3}{1-\beta}\right)^\gamma}{2} < \left(1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}\right)^\gamma.$$

Rearrange this condition as

$$\left(\frac{1}{\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta}\right)^\gamma + \left(\frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta\right)^\gamma < 2. \tag{A9}$$

As before, it can be shown that the expression on the left ("LHS") is a convex function of γ. Notice that $\lim_{\gamma\to\infty} LHS = +\infty > 2$. Morevoer, $LHS|_{\gamma=1} < 2$ since, with some algebraic manipulations, one can show that

$$\lambda^{1-\rho} < 1 \iff \frac{1}{\frac{1}{\lambda^{1-\rho}+\frac{\beta}{1-\beta}}+\beta} + \frac{1}{1+\lambda^{1-\rho}\beta+\frac{\beta^2}{1-\beta}}+\beta < 2.$$

Thus, there exists $\bar{\gamma} > 0$ such that $LHS > 2$ (RATL) if and only if $\gamma > \bar{\gamma}$. The result then follows from the fact that γ is increasing in α.

To conclude the proof, it remains to be shown that $\lim_{x \searrow 0} \bar{\alpha}_{\rho,\beta,x} = +\infty$. Both sides of (A3) are equal to $\left(\frac{1}{1-\beta}\right)^{\frac{1}{1-\rho}}$ when $\lambda = 1$. The derivative of the expression on the right (utility of the safe lottery) with respect to λ at $\lambda = 1$ is

$$\left(\frac{1}{1-\beta}\right)^{\frac{\rho}{1-\rho}} \beta^2. \tag{A10}$$

The derivative of the expression on the left (utility of the risky lottery) with respect to λ at $\lambda = 1$ is

$$\beta \frac{1+\beta^2}{2} \left(\frac{1}{1-\beta}\right)^{\frac{\rho}{1-\rho}}. \tag{A11}$$

With some algebraic manipulations, it can be shown that for any $\beta \in (0,1)$, the term in (A10) is lower than the one in (A11). $\square$

B.5 Proof of Proposition 5

The proof of the proposition will be presented in a series of lemmas. We start by obtaining a formula for the value of the kind of lotteries considered in this appendix:

Lemma 2. *In EZ, the value of lottery* $p \equiv \frac{1}{2} \times (x, 2) + \frac{1}{2} \times (y, t)$ *is*

$$U(p) = \left\{ (1-\beta) + \beta \left[\frac{\left[(1-\beta)(x+1)^{1-\rho} + \beta\right]^{\frac{1-\alpha}{1-\rho}} + \left\{1 + (1-\beta) \cdot \beta^{t-2} \left[(y+1)^{1-\rho} - 1\right]\right\}^{\frac{1-\alpha}{1-\rho}}}{2} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

Proof. For notational simplicity, let $z_1 \equiv 1+x$ and $z_2 \equiv 1+y$. We start by calculating the continuation value in period t, which is a constant stream of one in both lotteries: $V_t = 1$. Proceeding backwards, there are two possible states of the world, each with 50% chance: one in which the early prize is paid (in period 2), and one in which the late prize is paid (at $t > 2$).

When the early prize is paid, the individual still gets a constant stream of one for any $t > 2$, so that $V_3 = 1$. Plugging back in the utility at period 2, gives

$$V_2 = \left[(1-\beta) z_1^{1-\rho} + \beta\right]^{\frac{1}{1-\rho}}.$$

When the late prize is paid, we have

$$V_t = \left[(1-\beta) z_2^{1-\rho} + \beta\right]^{\frac{1}{1-\rho}}.$$

We claim that, for any $n = \{1, ..., t-2\}$,

$$V_{t-n} = \left[1 - (1-\beta)\,\beta^n \left(1 - z_2^{1-\rho}\right)\right]^{\frac{1}{1-\rho}},$$

so that, in particular,

$$V_2 = \left[1 - (1-\beta) \cdot \beta^{t-2} \left(1 - z_2^{1-\rho}\right)\right]^{\frac{1}{1-\rho}}.$$

To see this, we proceed inductively. At $t-1$, we have

$$V_{t-1} = \left\{(1-\beta) + \beta \left[E_t \left(V_t^{1-\alpha}\right)\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}} = \left\{(1-\beta) + \beta \left\{\left[(1-\beta)\, z_2^{1-\rho} + \beta\right]^{\frac{1-\alpha}{1-\rho}}\right\}^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}$$

$$= \left[(1-\beta)\left(1 + \beta z_2^{1-\rho}\right) + \beta^2\right]^{\frac{1}{1-\rho}}.$$

Moving back another period, gives:

$$V_{t-2} = \left\{1 - \beta + \beta \left[(1-\beta)\left(1 + \beta z_2^{1-\rho}\right) + \beta^2\right]\right\}^{\frac{1}{1-\rho}}$$

$$= \left\{(1-\beta)\left(1 + \beta + \beta^2 z_2^{1-\rho}\right) + \beta^3\right\}^{\frac{1}{1-\rho}}.$$

Suppose, in order to obtain the induction result, that

$$V_{t-n} = \left\{(1-\beta)\left(1 + \beta + ... + \beta^{n-1} + \beta^n z_2^{1-\rho}\right) + \beta^{n+1}\right\}^{\frac{1}{1-\rho}}.$$

Then,

$$V_{t-(n+1)} = \left[(1-\beta) + \beta \left(V_{t-n}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}} = \left[(1-\beta) + \beta \left[(1-\beta)\left(1 + \beta + ... + \beta^n z_2^{1-\rho}\right) + \beta^{n+1}\right]\right]^{\frac{1}{1-\rho}}$$

$$= \left[(1-\beta)\left(1 + \beta + \beta^2 + ... + \beta^n + \beta^{n+1} z_2^{1-\rho}\right) + \beta^{n+2}\right]^{\frac{1}{1-\rho}},$$

establishing the induction formula. Using the formula for the geometric progression, gives

$$V_{t-n} = \left[(1-\beta)\left(\frac{1-\beta^n}{1-\beta} + \beta^n z_2^{1-\rho}\right) + \beta^{n+1}\right]^{\frac{1}{1-\rho}} = \left[1 - \beta^n + \beta^{n+1} + (1-\beta)\left(\beta^n z_2^{1-\rho}\right)\right]^{\frac{1}{1-\rho}}$$

$$= \left[1 + (1-\beta)\,\beta^n \left(z_2^{1-\rho} - 1\right)\right]^{\frac{1}{1-\rho}},$$

which establishes our claim.

Since each of the two states happens with probability $\frac{1}{2}$, we have

$$E_1[V_2^{1-\alpha}] = \frac{\left[(1-\beta)\, z_1^{1-\rho} + \beta\right]^{\frac{1-\alpha}{1-\rho}} + \left[1 + (1-\beta) \cdot \beta^{t-2} \left(z_2^{1-\rho} - 1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2}.$$

Therefore, the value from the lottery equals

$$V_1 = \left\{ (1-\beta) + \beta \left\{ \frac{\left[(1-\beta)\, z_1^{1-\rho} + \beta \right]^{\frac{1-\alpha}{1-\rho}} + \left[1 + (1-\beta) \cdot \beta^{t-2} \left(z_2^{1-\rho} - 1 \right) \right]^{\frac{1-\alpha}{1-\rho}}}{2} \right\}^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}},$$

concluding the proof. $\square$

Next, we obtain necessary condition for EZ preferences not to be RSTL. By stationarity, it suffices to compare lotteries in which the early prize is paid at $t = 2$. That is, preferences are RSTL if and only if, for all $x > 0$ and all $\Delta \in \{1, 2, 3, ...\}$,

$$\frac{1}{2} \times (x, 2) + \frac{1}{2} \times (x, 2 + 2\Delta) \succeq (x, 2 + \Delta).$$

Let $\lambda \equiv \frac{c+x}{c} = 1 + x \in (1, +\infty)$. The value of the safe time lottery is

$$V^S \equiv (1-\beta)^{\frac{1}{1-\rho}} \cdot \left[\beta^{\Delta+1} \left(\lambda^{1-\rho} - 1 \right) + \frac{1}{1-\beta} \right]^{\frac{1}{1-\rho}}.$$

Using the formula from Lemma 2, we obtain the value of the risky time lottery:

$$\begin{aligned} V^R &= \left\{ (1-\beta) + \beta \left[\frac{\left[(1-\beta)\lambda^{1-\rho} + \beta \right]^{\frac{1-\alpha}{1-\rho}} + \left[1 + (1-\beta) \cdot \beta^{t_2 - 2} \left(\lambda^{1-\rho} - 1 \right) \right]^{\frac{1-\alpha}{1-\rho}}}{2} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}} \\ &= (1-\beta)^{\frac{1}{1-\rho}} \left\{ 1 + \beta \left[\frac{\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1 \right)^{\frac{1-\alpha}{1-\rho}} + \left[\frac{1}{1-\beta} + \beta^{2\Delta} \left(\lambda^{1-\rho} - 1 \right) \right]^{\frac{1-\alpha}{1-\rho}}}{2} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}. \end{aligned}$$

Therefore, preferences are RSTL if and only if

$$\left\{ 1 + \beta \left[\frac{\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1 \right)^{\frac{1-\alpha}{1-\rho}} + \left[\frac{1}{1-\beta} + \beta^{2\Delta} \left(\lambda^{1-\rho} - 1 \right) \right]^{\frac{1-\alpha}{1-\rho}}}{2} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}} \geq \left[\frac{1}{1-\beta} + \beta^{\Delta+1} \left(\lambda^{1-\rho} - 1 \right) \right]^{\frac{1}{1-\rho}} \tag{A12}$$

for all $\lambda > 1$ and all $\Delta \geq 1$.

To simplify notation, let $f(x) \equiv x^{\frac{1-\alpha}{1-\rho}}$. In the proofs below, we will repeatedly use the following inequality:

$$\frac{\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1 \right) + \left[\frac{1}{1-\beta} + \beta^{2\Delta} \left(\lambda^{1-\rho} - 1 \right) \right]}{2} \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{1}{1-\beta} + \beta^{\Delta} \left(\lambda^{1-\rho} - 1 \right) \iff \rho \left\{ \begin{array}{c} < \\ > \end{array} \right\} 1. \tag{A13}$$

We first verify that (A12) always holds when $\alpha < 1$.

Lemma 3. *Let $\alpha < 1$. Then, preferences are RSTL.*

Proof. There are three cases: (i) $\alpha \leq \rho < 1$, (ii) $\rho < \alpha < 1$, and (iii) $\alpha < 1 < \rho$.

Case (i): $\alpha \leq \rho < 1$. Since $\rho < 1$, equation (A12) can be written as

$$1+\beta\left[\frac{\left(\frac{1}{1-\beta}+\lambda^{1-\rho}-1\right)^{\frac{1-\alpha}{1-\rho}}+\left[\frac{1}{1-\beta}-\beta^{2\Delta}\left(\lambda^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}} \geq \frac{1}{1-\beta}+\beta^{\Delta+1}\left(\lambda^{1-\rho}-1\right).$$

Algebraic manipulations and the fact that $\frac{1-\rho}{1-\alpha} > 0$ allow us to rewrite this condition as

$$\frac{f\left(\frac{1}{1-\beta}+\lambda^{1-\rho}-1\right)+f\left(\frac{1}{1-\beta}-\beta^{2\Delta}\left(\lambda^{1-\rho}-1\right)\right)}{2} \geq f\left(\frac{1}{1-\beta}+\beta^{\Delta}\left(\lambda^{1-\rho}-1\right)\right).$$

Since f is increasing and convex when $\alpha < 1$ and $\rho < 1$, (A13) implies that this inequality is true.

Case (ii): $\rho < \alpha < 1$. Use $\rho < 1$ to rewrite equation (A12) as

$$\frac{\left(\frac{1}{1-\beta}+\lambda^{1-\rho}-1\right)^{\frac{1-\alpha}{1-\rho}}+\left[\frac{1}{1-\beta}+\beta^{2\Delta}\left(\lambda^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2} \geq \left[\frac{1}{1-\beta}+\beta^{\Delta}\left(\lambda^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}.$$

Rearrange this condition as

$$\left[\frac{\frac{1}{1-\beta}+\lambda^{1-\rho}-1}{\frac{1}{1-\beta}+\beta^{\Delta}\left(\lambda^{1-\rho}-1\right)}\right]^{\frac{1-\alpha}{1-\rho}}+\left[\frac{\frac{1}{1-\beta}+\beta^{2\Delta}\left(\lambda^{1-\rho}-1\right)}{\frac{1}{1-\beta}+\beta^{\Delta}\left(\lambda^{1-\rho}-1\right)}\right]^{\frac{1-\alpha}{1-\rho}} \geq 2$$

To simplify notation, perform the following change of variables: $\gamma \equiv \frac{1-\alpha}{1-\rho} \in (0,1)$, where $\gamma > 0$ since $\alpha < 1$ and $\rho < 1$ and $\gamma < 1$ follows from $\rho < \alpha$. After some algebraic manipulations, this inequality can be written as:

$$\left[\frac{1}{\frac{1-\beta^{\Delta}}{1+(1-\beta)(\lambda^{1-\rho}-1)}+\beta^{\Delta}}\right]^{\gamma}+\left[\frac{1}{\frac{1}{1-\beta^{\Delta}}+\beta^{\Delta}\cdot\left(\frac{1-\beta}{1-\beta^{\Delta}}\right)\left(\lambda^{1-\rho}-1\right)}+\beta^{\Delta}\right]^{\gamma} \geq 2.$$

It is straightforward to show that the expression on the left ("LHS") is a convex function of γ. Recall that $\gamma \in (0,1)$. Note that when $\gamma = 0$, the LHS equals 2. Since LHS is a convex function of γ, it suffices to show that its derivative with respect to γ at zero is positive. But note that

$$\left.\frac{dLHS}{d\gamma}\right|_{\gamma=0} = \ln\left[\frac{\frac{1}{\frac{1}{1-\beta^{\Delta}}+\beta^{\Delta}\cdot\left(\frac{1-\beta}{1-\beta^{\Delta}}\right)(\lambda^{1-\rho}-1)}+\beta^{\Delta}}{\frac{1-\beta^{\Delta}}{1+(1-\beta)(\lambda^{1-\rho}-1)}+\beta^{\Delta}}\right],$$

which, after some algebraic manipulations, can be shown to be strictly positive for any $\rho < 1$. Thus, $LHS > 2$ for all $\gamma \in (0, 1]$, establishing that (A12) holds.

Case (iii): $\alpha < 1 < \rho$. Since $\rho > 1$, equation (A12) becomes

$$\left[\frac{\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right)^{\frac{1-\alpha}{1-\rho}} + \left[\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho} - 1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}} \leq \frac{1}{1-\beta} + \beta^{\Delta}\left(\lambda^{1-\rho} - 1\right),$$

and, because $\frac{1-\rho}{1-\alpha} < 0$, this inequality holds if and only if:

$$\frac{f\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right) + f\left(\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho} - 1\right)\right)}{2} \geq f\left(\frac{1}{1-\beta} + \beta^{\Delta}\left(\lambda^{1-\rho} - 1\right)\right). \tag{A14}$$

Note that $\alpha < 1 < \rho$ implies that f is decreasing and convex. Since f is convex, Jensen's inequality implies:

$$\frac{f\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right) + f\left(\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho} - 1\right)\right)}{2} > f\left(\frac{\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right) + \left[\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho} - 1\right)\right]}{2}\right).$$

Then, by (A13) and the fact that f is decreasing, it follows that

$$\frac{f\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right) + f\left(\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho} - 1\right)\right)}{2} > f\left(\frac{1}{1-\beta} + \beta^{\Delta}\left(\lambda^{1-\rho} - 1\right)\right),$$

showing that inequality (A14) holds. $\square$

Lemma 4. *Let $\alpha \leq \rho$. Then, preferences are RSTL.*

Proof. By the previous lemma, the result is immediate when $\alpha < 1$. Therefore, let $\alpha > 1$ (which, by the statement of the lemma, requires $\rho > 1$). Using $\rho \geq \alpha > 1$, we can rewrite condition (A12) as

$$\frac{f\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right) + f\left(\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho} - 1\right)\right)}{2} \leq f\left(\frac{1}{1-\beta} + \beta^{\Delta}\left(\lambda^{1-\rho} - 1\right)\right),$$

which follows from condition (A13) and from the fact that f is increasing and concave when $\rho > \alpha > 1$. $\square$

Therefore, when either $\alpha < 1$ or $\alpha \leq \rho$, preferences must be RSTL. To show that a violation of RSTL implies a violation of SI, we must consider the remaining cases (where violations of RSTL are possible): $\alpha > 1 > \rho$ and $\alpha \geq \rho > 1$. The next two lemmas consider each of these cases separately.

Lemma 5. *Let $\alpha > 1 > \rho$. Then, preferences violate SI.*

Proof. Consider the lotteries $p_H \equiv \frac{1}{2}\times(H,2)+\frac{1}{2}\times(1,3)$ and $q_H \equiv \frac{1}{2}\times(1,2)+\frac{1}{2}\times(H,3)$. Note that to show that preferences violate SI, it suffices to show that $q_H \succ p_H$ for some $H > 1$ such that. By Lemma 2, the values of these lotteries are:

$$U(p_H) = \left\{(1-\beta) + \beta\left[\frac{\left[(1-\beta)(H+1)^{1-\rho}+\beta\right]^{\frac{1-\alpha}{1-\rho}} + \left[1+(1-\beta)\cdot\beta\left(2^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}.$$

and

$$U(q_H) = \left\{(1-\beta) + \beta\left[\frac{\left[(1-\beta)2^{1-\rho}+\beta\right]^{\frac{1-\alpha}{1-\rho}} + \left[1+(1-\beta)\cdot\beta\left((H+1)^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2}\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}.$$

Since $\alpha > 1 > \rho$, we find that $q_H \succ p_H$ if and only if

$$\left[(1-\beta)2^{1-\rho}+\beta\right]^{\frac{1-\alpha}{1-\rho}} - \left[(1-\beta)(H+1)^{1-\rho}+\beta\right]^{\frac{1-\alpha}{1-\rho}}$$
$$< \left[1+\beta(1-\beta)\left(2^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}} - \left\{1+\beta(1-\beta)\left[(H+1)^{1-\rho}-1\right]\right\}^{\frac{1-\alpha}{1-\rho}}.$$

Because $\frac{1-\alpha}{1-\rho} < 0$, as $H \nearrow +\infty$, the LHS converges to $\left[(1-\beta)2^{1-\rho}+\beta\right]^{\frac{1-\alpha}{1-\rho}}$, whereas the RHS converges to $\left[1+\beta(1-\beta)\left(2^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}$. Thus, there exists $\bar{H}$ such that this inequality holds for all $H > \bar{H}$ if

$$\left[(1-\beta)2^{1-\rho}+\beta\right]^{\frac{1-\alpha}{1-\rho}} < \left[1+\beta(1-\beta)\left(2^{1-\rho}-1\right)\right]^{\frac{1-\alpha}{1-\rho}}.$$

Use the fact that $\frac{1-\alpha}{1-\rho} < 0$ to rewrite this inequality as:

$$(1-\beta)2^{1-\rho}+\beta > 1+\beta(1-\beta)\left(2^{1-\rho}-1\right) \iff \left(2^{1-\rho}-1\right)(1-\beta) > 0,$$

which is always true since $\rho < 1$. $\square$

Lemma 6. *Let $\alpha > \rho > 1$. If $\frac{\rho-1}{\alpha-\rho} < 1 - \frac{\ln[1-(1-\beta)\beta]}{\ln\beta}$, then preferences violate SI.*

Proof. We claim that there exist H and $L < H$ such that

$$\frac{1}{2}\times(H,2)+\frac{1}{2}\times(L,\ 3) \prec \frac{1}{2}\times(L,2)+\frac{1}{2}\times(H,\ 2+3) \tag{A15}$$

if and only if

$$\frac{\rho-1}{\alpha-\rho} < 1 - \frac{\ln\left[1-(1-\beta)\beta\right]}{\ln\beta}. \tag{A16}$$

For each fixed z_H and z_L, consider the following lotteries

$$p_{H,L} \equiv \frac{1}{2} \times (z_H - 1, 2) + \frac{1}{2} \times (z_L - 1, 3),$$

and

$$q_{H,L} \equiv \frac{1}{2} \times (z_L - 1, 2) + \frac{1}{2} \times (z_H - 1, 3).$$

By Lemma 2, the values of lotteries $p_{H,L}$ and $q_{H,L}$ are:

$$U(p_{H,L}) = \left\{ (1-\beta) + \beta \left[\frac{\left[(1-\beta) z_H^{1-\rho} + \beta\right]^{\frac{1-\alpha}{1-\rho}} + \left[1 + \beta(1-\beta)\left(z_L^{1-\rho} - 1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}},$$

$$U(q_{H,L}) = \left\{ (1-\beta) + \beta \left[\frac{\left[(1-\beta) z_L^{1-\rho} + \beta\right]^{\frac{1-\alpha}{1-\rho}} + \left[1 + \beta(1-\beta)\left(z_H^{1-\rho} - 1\right)\right]^{\frac{1-\alpha}{1-\rho}}}{2} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}.$$

For notational simplicity, let $\mu_H \equiv 1 - z_H^{1-\rho}$ and $\mu_L \equiv 1 - z_L^{1-\rho}$ and note that $0 < \mu_L < \mu_H < 1$ (since $1 < z_L < z_H$ and $\rho > 1$). Using the fact that $\frac{1}{1-\rho} < 0$ and $\frac{1-\alpha}{1-\rho} > 0$, it follows that $q_{H,L} \succ p_{H,L}$ if and only if

$$[(1-\beta)(1-\mu_L) + \beta]^{\frac{1-\alpha}{1-\rho}} + [1 - \beta(1-\beta)\mu_H]^{\frac{1-\alpha}{1-\rho}} < [(1-\beta)(1-\mu_H) + \beta]^{\frac{1-\alpha}{1-\rho}} + [1 - \beta(1-\beta)\mu_L]^{\frac{1-\alpha}{1-\rho}}.$$

Let $\phi(\mu) \equiv [(1-\beta)(1-\mu) + \beta]^{\frac{1-\alpha}{1-\rho}} - [1 - \beta(1-\beta)\mu]^{\frac{1-\alpha}{1-\rho}}$, and note that, by the previous inequality, there exists $z_H > z_L > 1$ such that $q_{H,L} \succ p_{H,L}$ if and only if $\phi(\mu_H) > \phi(\mu_L)$ for some μ_H and μ_L with $0 < \mu_L < \mu_H < 1$. That is, $q_{H,L} \succ p_{H,L}$ for some $z_H > z_L > 1$ if and only if $\phi(\cdot)$ is not weakly decreasing in the interval $(0,1)$, which, because $\phi(\cdot)$ is differentiable, is true if and only if $\phi'(\mu) > 0$ for some μ.

Differentiating $\phi(\cdot)$, gives:

$$\phi'(\mu) = \left(\frac{1-\alpha}{1-\rho}\right)(1-\beta)\left\{\beta\,[1 - \beta(1-\beta)\mu]^{\frac{1-\alpha}{1-\rho}-1} - [(1-\beta)(1-\mu) + \beta]^{\frac{1-\alpha}{1-\rho}-1}\right\},$$

so that $\phi'(\mu) > 0$ if and only if

$$\beta\,[1 - \beta(1-\beta)\mu]^{\frac{1-\alpha}{1-\rho}-1} > [(1-\beta)(1-\mu) + \beta]^{\frac{1-\alpha}{1-\rho}-1}.$$

Notice that the terms inside the brackets are positive (because $\mu \in (0,1)$), so we can simplify this condition as

$$\mu > \frac{1}{1-\beta} \cdot \frac{1 - \beta^{\frac{\rho-1}{\alpha-\rho}}}{1 - \beta^{\frac{\rho-1}{\alpha-\rho}+1}}$$

for some $\mu \in (0,1)$. But this is true if and only if the inequality holds for $\mu = 1$:

$$1 > \frac{1}{1-\beta} \cdot \frac{1-\beta^{\frac{\rho-1}{\alpha-\rho}}}{1-\beta^{\frac{\rho-1}{\alpha-\rho}+1}},$$

which can be rearranged as

$$\beta^{\frac{\rho-1}{\alpha-\rho}} > \frac{\beta}{1-(1-\beta)\beta}.$$

Taking logs of both sides gives the following necessary and sufficient condition for (A15):

$$\frac{\rho-1}{\alpha-\rho}\ln\beta > \ln\beta - \ln\left[1-(1-\beta)\beta\right],$$

which, because $\ln\beta < 0$, can be rearranged as

$$\frac{\rho-1}{\alpha-\rho} < 1 - \frac{\ln\left[1-(1-\beta)\beta\right]}{\ln\beta},$$

which is condition (A16). □

Lemma 7. *Let $\alpha \geq \rho > 1$. Preferences are RSTL if and only if*

$$\frac{\left(\frac{1}{1-\beta} - y\right)^{\frac{1-\alpha}{1-\rho}} + \left(\frac{1}{1-\beta} - \beta^{2\Delta} y\right)^{\frac{1-\alpha}{1-\rho}}}{2} \leq \left(\frac{1}{1-\beta} - \beta^{\Delta} y\right)^{\frac{1-\alpha}{1-\rho}} \tag{A17}$$

for all $y \in (0,1)$ and all $\Delta \in \{1,2,3,...\}$.

Proof. Let $\gamma \equiv \frac{1-\alpha}{1-\rho} > 0$. By (A12) (and the fact that $\rho > 1$), preferences are RSTL if and only if

$$\frac{\left(\frac{1}{1-\beta} + \lambda^{1-\rho} - 1\right)^{\gamma} + \left[\frac{1}{1-\beta} + \beta^{2\Delta}\left(\lambda^{1-\rho}-1\right)\right]^{\gamma}}{2} \leq \left[\frac{1}{1-\beta} + \beta^{\Delta}\left(\lambda^{1-\rho}-1\right)\right]^{\gamma}$$

for all $\lambda > 1$ and all $\Delta \geq 1$. Let $y \equiv 1 - \lambda^{1-\rho}$ and notice that $y \in (0,1)$ (since $\lambda \in (1,+\infty)$ and $\rho > 1$). Thus, we can rewrite the RSTL condition as

$$\frac{\left(\frac{1}{1-\beta} - y\right)^{\gamma} + \left(\frac{1}{1-\beta} - \beta^{2\Delta} y\right)^{\gamma}}{2} \leq \left(\frac{1}{1-\beta} - \beta^{\Delta} y\right)^{\gamma}$$

for all $y \in (0,1)$. □

Lemma 8. *Let $\alpha \geq \rho > 1$ and suppose preferences violate RSTL. Then preferences violate SI.*

Proof. Let $\gamma \equiv \frac{1-\alpha}{1-\rho} > 1$. We claim that for each fixed y, β, and Δ, there exists a threshold $\bar{\gamma}_{y,\beta,\Delta}$ such that preferences are RSTL if and only if $\gamma \geq \bar{\gamma}_{y,\beta,\Delta}$, which, by the previous lemma, is equivalent to

$$\frac{\left(\frac{1}{1-\beta} - y\right)^{\gamma} + \left(\frac{1}{1-\beta} - \beta^{2\Delta} y\right)^{\gamma}}{2} > \left(\frac{1}{1-\beta} - \beta^{\Delta} y\right)^{\gamma} \iff \gamma < \bar{\gamma}_{y,\beta}. \tag{A18}$$

To see this, rearrange (A18) as

$$\left(\frac{\frac{1}{1-\beta} - y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)^{\gamma} + \left(\frac{\frac{1}{1-\beta} - \beta^{2\Delta} y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)^{\gamma} > 2. \tag{A19}$$

Notice first that the expression on the LHS of (A19) is a convex function of γ, since

$$\frac{d^2}{d\gamma^2} LHS = \left(\frac{\frac{1}{1-\beta} - y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)^{\gamma} \cdot \left[\ln\left(\frac{\frac{1}{1-\beta} - y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)\right]^2 + \left(\frac{\frac{1}{1-\beta} - \beta^{2\Delta} y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)^{\gamma} \cdot \left[\ln\left(\frac{\frac{1}{1-\beta} - \beta^{2\Delta} y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)\right]^2 > 0.$$

Algebraic manipulations establish that (A19) fails for $\gamma = 1$. Moreover, (A19) is always true for γ large enough, since

$$\lim_{\gamma \to \infty} \left(\frac{\frac{1}{1-\beta} - y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)^{\gamma} + \left(\frac{\frac{1}{1-\beta} - \beta^{2\Delta} y}{\frac{1}{1-\beta} - \beta^{\Delta} y}\right)^{\gamma} = +\infty > 2.$$

Therefore, there exists a unique $\bar{\gamma}_{\beta,y,\Delta} > 1$ such that the inequality holds if and only if $\gamma > \bar{\gamma}_{\beta,y,\Delta}$.

Recall from Lemma 6 that preferences violate SI if

$$\frac{\rho - 1}{\alpha - \rho} < 1 - \frac{\ln\left[1 - (1-\beta)\beta\right]}{\ln \beta}.$$

Since $\gamma - 1 = \frac{\alpha-\rho}{\rho-1}$, this condition can be written as

$$\frac{1}{\gamma - 1} < 1 - \frac{\ln\left[1 - (1-\beta)\beta\right]}{\ln \beta},$$

which can be further simplified as

$$\gamma > \frac{\ln\left[1 - (1-\beta)\beta\right] - 2\ln\beta}{\ln\left[1 - (1-\beta)\beta\right] - \ln\beta}.$$

Therefore, preferences violate RSTL if and only if $\gamma \geq \bar{\gamma}_{\beta,y,\Delta}$, whereas they violate SI if $\gamma \geq \frac{\ln[1-(1-\beta)\beta] - 2\ln\beta}{\ln[1-(1-\beta)\beta] - \ln\beta}$. To conclude the proof, it suffices to show that the cutoff for RSTL violations is higher than the (sufficient) cutoff for SI violations:

$$\bar{\gamma}_{\beta,y,\Delta} \geq \frac{\ln\left[1 - (1-\beta)\beta\right] - 2\ln\beta}{\ln\left[1 - (1-\beta)\beta\right] - \ln\beta}.$$

Recall that $\bar{\gamma}_{\beta,y,\Delta}$ solves:

$$\left(\frac{\frac{1}{1-\beta}-y}{\frac{1}{1-\beta}-\beta^{\Delta}y}\right)^{\gamma}+\left(\frac{\frac{1}{1-\beta}-\beta^{2\Delta}y}{\frac{1}{1-\beta}-\beta^{\Delta}y}\right)^{\gamma}=2.$$

Note that LHS is convex, $LHS(1)<2$ and $LHS(\infty)>2$. Thus, we need to show that

$$\left.\left(\frac{\frac{1}{1-\beta}-y}{\frac{1}{1-\beta}-\beta^{\Delta}y}\right)^{\gamma}+\left(\frac{\frac{1}{1-\beta}-\beta^{2\Delta}y}{\frac{1}{1-\beta}-\beta^{\Delta}y}\right)^{\gamma}\right|_{\gamma=\frac{\ln[1-(1-\beta)\beta]-2\ln\beta}{\ln[1-(1-\beta)\beta]-\ln\beta}}<2.$$

Note that

$$\frac{\ln\left[1-(1-\beta)\beta\right]-2\ln\beta}{\ln\left[1-(1-\beta)\beta\right]-\ln\beta}<2.$$

So, it suffices to show that

$$\left(\frac{\frac{1}{1-\beta}-y}{\frac{1}{1-\beta}-\beta^{\Delta}y}\right)^{2}+\left(\frac{\frac{1}{1-\beta}-\beta^{2\Delta}y}{\frac{1}{1-\beta}-\beta^{\Delta}y}\right)^{2}<2$$

for all for all $y\in(0,1)$ and all Δ,β. Rearrange this expression as

$$\frac{\left(\frac{1}{1-\beta}-y\right)^{2}+\left(\frac{1}{1-\beta}-\beta^{2\Delta}y\right)^{2}}{2}<\left(\frac{1}{1-\beta}-\beta^{\Delta}y\right)^{2}.$$

With some algebraic manipulations, this inequality can be rewritten as

$$\left(1+\beta^{\Delta}\right)^{2}<\frac{2}{1-\beta}$$

for all $y\in(0,1)$ and all $\Delta=1,2,3....$ Since the LHS is decreasing in Δ (because $\beta<1$), it suffices to verify this condition at $\Delta=1$, where we have

$$(1+\beta)^{2}<\frac{2}{1-\beta}\iff 0<1-\beta+\beta^{2}+\beta^{3}.$$

Let $\xi(\beta)\equiv 1-\beta+\beta^{2}+\beta^{3}$ and notice that

$$\xi'(\beta)=-1+2\beta+3\beta^{2},$$

which has roots $\beta=-1$ and $\beta=\frac{1}{3}$. Moreover, ξ is convex at $\beta\in[0,1]$ since $\xi''(\beta)=2+6\beta>0$. Therefore, $\xi'(\beta)<0$ for $\beta\in[0,\frac{1}{3})$ and $\xi'(\beta)>0$ for $\beta\in(\frac{1}{3},1]$, showing that ξ has a minimum at $\beta=\frac{1}{3}$:

$$\xi(\beta)\geq\xi(\frac{1}{3})=1-\frac{1}{3}+\left(\frac{1}{3}\right)^{2}+\left(\frac{1}{3}\right)^{3}>0,$$

concluding the proof. $\square$

Combining the results from the lemmas above, it follows that preferences that satisfy SI must be RSTL, concluding the proof of the proposition. To see this, recall that:

- When $\alpha < 1$, preferences are always RSTL, regardless of whether they satisfy SI (Lemma 3).
- When$\rho \geq \alpha$, preferences are always RSTL, regardless of whether they satisfy SI (Lemma 4).
- When $\alpha > 1 > \rho$, SI never holds (Lemma 5).
- When $\alpha > \rho > 1$, SI holds if γ is below a threshold that is lower than the threshold for RSTL ($\frac{\ln[1-(1-\beta)\beta]-2\ln\beta}{\ln[1-(1-\beta)\beta]-\ln\beta} < \bar{\gamma}_{\beta,y,\Delta}$), so that SI implies RSTL (Lemma 8).

■

C Questionnaire and Instructions of the Experiment

The following is an example of the questionnaire used in the experiment, in the Short treatment, followed by the instructions used in the experiment. Page breaks in the questionnaire are similar to those used in the actual experiment.

Questionnaire Part 1

QUESTIONNAIRE – PART I

Please indicate your lab id: _________

Please answer each of the following questions by checking the box of the preferred option.
If the question is selected for payment, you will get the payment specified above the question, with a payment date based on your choice and, in some cases, on chance.

Question 1

Payment: **$20.** Payment date:

Option A			Option B
2 weeks	☐	☐	75% chance of 1 week 25% chance of 5 weeks

Question 2

Payment: **$15.** Payment date:

Option A			Option B
3 weeks	☐	☐	50% chance of 1 week 50% chance of 5 weeks

Question 3

Payment: **$10.** Payment date:

Option A			Option B
2 weeks	☐	☐	50% chance of 1 week 50% chance of 3 weeks

Question 4

Payment: **$20.** Payment date:

Option A			Option B
50% chance of 2 weeks 50% chance of 3 weeks	☐	☐	75% chance of 2 weeks 25% chance of 4 weeks

Question 5

Payment: **$10.** Payment date:

Option A			**Option B**
50% chance of 2 weeks 50% chance of 5 weeks	☐	☐	75% chance of 3 weeks 25% chance of 5 weeks

Questionnaire Part 2

QUESTIONNAIRE – PART II

Please indicate your lab id: _________

Please answer each of the following questions by checking the box of the preferred option for every row:

Question 6

Row	**Option A**			**Option B**
1	$10.00 today	☐	☐	$10.00 in 2 weeks
2	$10.00 today	☐	☐	$10.25 in 2 weeks
3	$10.00 today	☐	☐	$10.50 in 2 weeks
4	$10.00 today	☐	☐	$10.75 in 2 weeks
5	$10.00 today	☐	☐	$11.00 in 2 weeks
6	$10.00 today	☐	☐	$11.25 in 2 weeks
7	$10.00 today	☐	☐	$11.50 in 2 weeks
8	$10.00 today	☐	☐	$11.75 in 2 weeks
9	$10.00 today	☐	☐	$12.00 in 2 weeks
10	$10.00 today	☐	☐	$12.25 in 2 weeks
11	$10.00 today	☐	☐	$12.50 in 2 weeks
12	$10.00 today	☐	☐	$12.75 in 2 weeks
13	$10.00 today	☐	☐	$13.00 in 2 weeks
14	$10.00 today	☐	☐	$13.25 in 2 weeks
15	$10.00 today	☐	☐	$13.50 in 2 weeks
16	$10.00 today	☐	☐	$13.75 in 2 weeks
17	$10.00 today	☐	☐	$14.00 in 2 weeks
18	$10.00 today	☐	☐	$14.25 in 2 weeks
19	$10.00 today	☐	☐	$14.50 in 2 weeks
20	$10.00 today	☐	☐	$14.75 in 2 weeks
21	$10.00 today	☐	☐	$15.00 in 2 weeks

Question 7

Row	Option A			Option B
1	\$10.00 in 1 week	☐	☐	\$10.00 in 2 weeks
2	\$10.00 in 1 week	☐	☐	\$10.25 in 2 weeks
3	\$10.00 in 1 week	☐	☐	\$10.50 in 2 weeks
4	\$10.00 in 1 week	☐	☐	\$10.75 in 2 weeks
5	\$10.00 in 1 week	☐	☐	\$11.00 in 2 weeks
6	\$10.00 in 1 week	☐	☐	\$11.25 in 2 weeks
7	\$10.00 in 1 week	☐	☐	\$11.50 in 2 weeks
8	\$10.00 in 1 week	☐	☐	\$11.75 in 2 weeks
9	\$10.00 in 1 week	☐	☐	\$12.00 in 2 weeks
10	\$10.00 in 1 week	☐	☐	\$12.25 in 2 weeks
11	\$10.00 in 1 week	☐	☐	\$12.50 in 2 weeks
12	\$10.00 in 1 week	☐	☐	\$12.75 in 2 weeks
13	\$10.00 in 1 week	☐	☐	\$13.00 in 2 weeks
14	\$10.00 in 1 week	☐	☐	\$13.25 in 2 weeks
15	\$10.00 in 1 week	☐	☐	\$13.50 in 2 weeks
16	\$10.00 in 1 week	☐	☐	\$13.75 in 2 weeks
17	\$10.00 in 1 week	☐	☐	\$14.00 in 2 weeks
18	\$10.00 in 1 week	☐	☐	\$14.25 in 2 weeks
19	\$10.00 in 1 week	☐	☐	\$14.50 in 2 weeks
20	\$10.00 in 1 week	☐	☐	\$14.75 in 2 weeks
21	\$10.00 in 1 week	☐	☐	\$15.00 in 2 weeks

Question 8

Row	Option A			Option B
1	$10.00 in 1 week	☐	☐	$10.00 in 3 weeks
2	$10.00 in 1 week	☐	☐	$10.25 in 3 weeks
3	$10.00 in 1 week	☐	☐	$10.50 in 3 weeks
4	$10.00 in 1 week	☐	☐	$10.75 in 3 weeks
5	$10.00 in 1 week	☐	☐	$11.00 in 3 weeks
6	$10.00 in 1 week	☐	☐	$11.25 in 3 weeks
7	$10.00 in 1 week	☐	☐	$11.50 in 3 weeks
8	$10.00 in 1 week	☐	☐	$11.75 in 3 weeks
9	$10.00 in 1 week	☐	☐	$12.00 in 3 weeks
10	$10.00 in 1 week	☐	☐	$12.25 in 3 weeks
11	$10.00 in 1 week	☐	☐	$12.50 in 3 weeks
12	$10.00 in 1 week	☐	☐	$12.75 in 3 weeks
13	$10.00 in 1 week	☐	☐	$13.00 in 3 weeks
14	$10.00 in 1 week	☐	☐	$13.25 in 3 weeks
15	$10.00 in 1 week	☐	☐	$13.50 in 3 weeks
16	$10.00 in 1 week	☐	☐	$13.75 in 3 weeks
17	$10.00 in 1 week	☐	☐	$14.00 in 3 weeks
18	$10.00 in 1 week	☐	☐	$14.25 in 3 weeks
19	$10.00 in 1 week	☐	☐	$14.50 in 3 weeks
20	$10.00 in 1 week	☐	☐	$14.75 in 3 weeks
21	$10.00 in 1 week	☐	☐	$15.00 in 3 weeks

Question 9

Row	Option A			Option B
1	$10.00 in 1 week	☐	☐	$10.00 in 4 weeks
2	$10.00 in 1 week	☐	☐	$10.25 in 4 weeks
3	$10.00 in 1 week	☐	☐	$10.50 in 4 weeks
4	$10.00 in 1 week	☐	☐	$10.75 in 4 weeks
5	$10.00 in 1 week	☐	☐	$11.00 in 4 weeks
6	$10.00 in 1 week	☐	☐	$11.25 in 4 weeks
7	$10.00 in 1 week	☐	☐	$11.50 in 4 weeks
8	$10.00 in 1 week	☐	☐	$11.75 in 4 weeks
9	$10.00 in 1 week	☐	☐	$12.00 in 4 weeks
10	$10.00 in 1 week	☐	☐	$12.25 in 4 weeks
11	$10.00 in 1 week	☐	☐	$12.50 in 4 weeks
12	$10.00 in 1 week	☐	☐	$12.75 in 4 weeks
13	$10.00 in 1 week	☐	☐	$13.00 in 4 weeks
14	$10.00 in 1 week	☐	☐	$13.25 in 4 weeks
15	$10.00 in 1 week	☐	☐	$13.50 in 4 weeks
16	$10.00 in 1 week	☐	☐	$13.75 in 4 weeks
17	$10.00 in 1 week	☐	☐	$14.00 in 4 weeks
18	$10.00 in 1 week	☐	☐	$14.25 in 4 weeks
19	$10.00 in 1 week	☐	☐	$14.50 in 4 weeks
20	$10.00 in 1 week	☐	☐	$14.75 in 4 weeks
21	$10.00 in 1 week	☐	☐	$15.00 in 4 weeks

Question 10

Payment: **$25.** Payment date:

Row	Option A		Option B			
1	In 3 weeks	☐	☐	0%	chance of	2 weeks
				100%	chance of	5 weeks
2	In 3 weeks	☐	☐	5%	chance of	2 weeks
				95%	chance of	5 weeks
3	In 3 weeks	☐	☐	10%	chance of	2 weeks
				90%	chance of	5 weeks
4	In 3 weeks	☐	☐	15%	chance of	2 weeks
				85%	chance of	5 weeks
5	In 3 weeks	☐	☐	20%	chance of	2 weeks
				80%	chance of	5 weeks
6	In 3 weeks	☐	☐	25%	chance of	2 weeks
				75%	chance of	5 weeks
7	In 3 weeks	☐	☐	30%	chance of	2 weeks
				70%	chance of	5 weeks
8	In 3 weeks	☐	☐	35%	chance of	2 weeks
				65%	chance of	5 weeks
9	In 3 weeks	☐	☐	40%	chance of	2 weeks
				60%	chance of	5 weeks
10	In 3 weeks	☐	☐	45%	chance of	2 weeks
				55%	chance of	5 weeks
11	In 3 weeks	☐	☐	50%	chance of	2 weeks
				50%	chance	5

Question 11

Payment: **$25.** Payment date:

Row	Option A			Option B		
1	In 2 weeks	☐	☐	0%	chance of	1 week
				100%	chance of	5 weeks
2	In 2 weeks	☐	☐	5%	chance of	1 week
				95%	chance of	5 weeks
3	In 2 weeks	☐	☐	10%	chance of	1 week
				90%	chance of	5 weeks
4	In 2 weeks	☐	☐	15%	chance of	1 week
				85%	chance of	5 weeks
5	In 2 weeks	☐	☐	20%	chance of	1 week
				80%	chance of	5 weeks
6	In 2 weeks	☐	☐	25%	chance of	1 week
				75%	chance of	5 weeks
7	In 2 weeks	☐	☐	30%	chance of	1 week
				70%	chance of	5 weeks
8	In 2 weeks	☐	☐	35%	chance of	1 week
				65%	chance of	5 weeks
9	In 2 weeks	☐	☐	40%	chance of	1 week
				60%	chance of	5 weeks
10	In 2 weeks	☐	☐	45%	chance of	1 week
				55%	chance of	5 weeks
11	In 2 weeks	☐	☐	50%	chance of	1 week
				50%	chance of	5 weeks
12	In 2 weeks	☐	☐	55%	chance of	1 week
				45%	chance of	5 weeks
13	In 2 weeks	☐	☐	60%	chance of	1 week
				40%	chance of	5 weeks
14	In 2 weeks	☐	☐	65%	chance of	1 week
				35%	chance of	5 weeks
15	In 2 weeks	☐	☐	70%	chance of	1 week
				30%	chance of	5 weeks
16	In 2 weeks	☐	☐	75%	chance of	1 week
				25%	chance of	5 weeks
17	In 2 weeks	☐	☐	80%	chance of	1 week
				20%	chance of	5 weeks
18	In 2 weeks	☐	☐	85%	chance of	1 week
				15%	chance of	5 weeks
19	In 2 weeks	☐	☐	90%	chance of	1 week
				10%	chance of	5 weeks
20	In 2 weeks	☐	☐	95%	chance of	1 week

Questionnaire Part 3

QUESTIONNAIRE – PART III

Please indicate your lab id: _________

Please answer each of the following questions by checking the box of the preferred option for every row:

Question 12

Row	Option A			Option B		
1	$15	☐	☐	0%	chance of	$20
				100%	chance of	$8
2	$15	☐	☐	5%	chance of	$20
				95%	chance of	$8
3	$15	☐	☐	10%	chance of	$20
				90%	chance of	$8
4	$15	☐	☐	15%	chance of	$20
				85%	chance of	$8
5	$15	☐	☐	20%	chance of	$20
				80%	chance of	$8
6	$15	☐	☐	25%	chance of	$20
				75%	chance of	$8
7	$15	☐	☐	30%	chance of	$20
				70%	chance of	$8
8	$15	☐	☐	35%	chance of	$20
				65%	chance of	$8
9	$15	☐	☐	40%	chance of	$20
				60%	chance of	$8
10	$15	☐	☐	45%	chance of	$20
				55%	chance of	$8
11	$15	☐	☐	50%	chance of	$20
				50%	chance	$8

Question 13

Row	**Option A**					**Option B**		
1	50%	chance of	$15	☐	☐	0%	chance of	$20
	50%	chance of	$8			100%	chance of	$8
2	50%	chance of	$15	☐	☐	5%	chance of	$20
	50%	chance of	$8			95%	chance of	$8
3	50%	chance of	$15	☐	☐	10%	chance of	$20
	50%	chance of	$8			90%	chance of	$8
4	50%	chance of	$15	☐	☐	15%	chance of	$20
	50%	chance of	$8			85%	chance of	$8
5	50%	chance of	$15	☐	☐	20%	chance of	$20
	50%	chance of	$8			80%	chance of	$8
6	50%	chance of	$15	☐	☐	25%	chance of	$20
	50%	chance of	$8			75%	chance of	$8
7	50%	chance of	$15	☐	☐	30%	chance of	$20
	50%	chance of	$8			70%	chance of	$8
8	50%	chance of	$15	☐	☐	35%	chance of	$20
	50%	chance of	$8			65%	chance of	$8
9	50%	chance of	$15	☐	☐	40%	chance of	$20
	50%	chance of	$8			60%	chance of	$8
10	50%	chance of	$15	☐	☐	45%	chance of	$20
	50%	chance of	$8			55%	chance of	$8
11	50%	chance of	$15	☐	☐	50%	chance of	$20
	50%	chance	$8			50%	chance	$8

Question 14

Row	**Option A**					**Option B**		
1	20%	chance of	$15	☐	☐	0%	chance of	$20
	80%	chance of	$8			100%	chance of	$8
2	20%	chance of	$15	☐	☐	5%	chance of	$20
	80%	chance of	$8			95%	chance of	$8
3	20%	chance of	$15	☐	☐	10%	chance of	$20
	80%	chance of	$8			90%	chance of	$8
4	20%	chance of	$15	☐	☐	15%	chance of	$20
	80%	chance of	$8			85%	chance of	$8
5	20%	chance of	$15	☐	☐	20%	chance of	$20
	80%	chance of	$8			80%	chance of	$8
6	20%	chance of	$15	☐	☐	25%	chance of	$20
	80%	chance of	$8			75%	chance of	$8
7	20%	chance of	$15	☐	☐	30%	chance of	$20
	80%	chance of	$8			70%	chance of	$8
8	20%	chance of	$15	☐	☐	35%	chance of	$20
	80%	chance of	$8			65%	chance of	$8
9	20%	chance of	$15	☐	☐	40%	chance of	$20
	80%	chance of	$8			60%	chance of	$8
10	20%	chance of	$15	☐	☐	45%	chance of	$20
	80%	chance of	$8			55%	chance of	$8
11	20%	chance of	$15	☐	☐	50%	chance of	$20
	80%	chance	$8			50%	chance	$8

Question 15

Row	**Option A**			**Option B**
1	$20	☐	☐	0% chance of $30 100% chance of $5
2	$20	☐	☐	5% chance of $30 95% chance of $5
3	$20	☐	☐	10% chance of $30 90% chance of $5
4	$20	☐	☐	15% chance of $30 85% chance of $5
5	$20	☐	☐	20% chance of $30 80% chance of $5
6	$20	☐	☐	25% chance of $30 75% chance of $5
7	$20	☐	☐	30% chance of $30 70% chance of $5
8	$20	☐	☐	35% chance of $30 65% chance of $5
9	$20	☐	☐	40% chance of $30 60% chance of $5
10	$20	☐	☐	45% chance of $30 55% chance of $5
11	$20	☐	☐	50% chance of $30 50% chance $5

Question 16

Row	**Option A**				**Option B**			
1	50%	chance of	$20	☐	☐	0%	chance of	$30
	50%	chance of	$5			100%	chance of	$3
2	50%	chance of	$20	☐	☐	5%	chance of	$30
	50%	chance of	$5			95%	chance of	$3
3	50%	chance of	$20	☐	☐	10%	chance of	$30
	50%	chance of	$5			90%	chance of	$3
4	50%	chance of	$20	☐	☐	15%	chance of	$30
	50%	chance of	$5			85%	chance of	$3
5	50%	chance of	$20	☐	☐	20%	chance of	$30
	50%	chance of	$5			80%	chance of	$3
6	50%	chance of	$20	☐	☐	25%	chance of	$30
	50%	chance of	$5			75%	chance of	$3
7	50%	chance of	$20	☐	☐	30%	chance of	$30
	50%	chance of	$5			70%	chance of	$3
8	50%	chance of	$20	☐	☐	35%	chance of	$30
	50%	chance of	$5			65%	chance of	$3
9	50%	chance of	$20	☐	☐	40%	chance of	$30
	50%	chance of	$5			60%	chance of	$3
10	50%	chance of	$20	☐	☐	45%	chance of	$30
	50%	chance of	$5			55%	chance of	$3
11	50%	chance of	$20	☐	☐	50%	chance of	$30
	50%	chance	$5			50%	chance	$3

Question 17

Row	**Option A**					**Option B**		
1	10%	chance of	$20	☐	☐	0%	chance of	$30
	90%	chance of	$5			100%	chance of	$3
2	10%	chance of	$20	☐	☐	5%	chance of	$30
	90%	chance of	$5			95%	chance of	$3
3	10%	chance of	$20	☐	☐	10%	chance of	$30
	90%	chance of	$5			90%	chance of	$3
4	10%	chance of	$20	☐	☐	15%	chance of	$30
	90%	chance of	$5			85%	chance of	$3
5	10%	chance of	$20	☐	☐	20%	chance of	$30
	90%	chance of	$5			80%	chance of	$3
6	10%	chance of	$20	☐	☐	25%	chance of	$30
	90%	chance of	$5			75%	chance of	$3
7	10%	chance of	$20	☐	☐	30%	chance of	$30
	90%	chance of	$5			70%	chance of	$3
8	10%	chance of	$20	☐	☐	35%	chance of	$30
	90%	chance of	$5			65%	chance of	$3
9	10%	chance of	$20	☐	☐	40%	chance of	$30
	90%	chance of	$5			60%	chance of	$3
10	10%	chance of	$20	☐	☐	45%	chance of	$30
	90%	chance of	$5			55%	chance of	$3
11	10%	chance of	$20	☐	☐	50%	chance of	$30
	90%	chance	$5			50%	chance	$3

Instructions: General Instructions

INSTRUCTIONS

OVERVIEW

This is an experiment in the economics of decision-making. The instructions are simple, and if you follow them carefully you may earn a considerable amount of money.

There are **three** parts in this experiment. In each part, you will be asked to answer some questions in a questionnaire that we will distribute. Please answer **all** the questions. We will hand out specific instructions for each part before it begins, and we will read these instructions aloud and answer any question you may have. After you have filled out the questionnaire of a given part, please put it to the side to indicate that you are done.

In the first page of all questionnaires you will be asked to write your '**lab id**. This is the identifier given to you by the Wharton Behavioral Lab. Please write this number on all questionnaires: this will be essential to ensure appropriate payment.

After you have answered all the questions in the experiment, we will ask you a brief survey with some additional questions about your experience.

Notice that in this experiment **there are no right or wrong answers**. We are interested in studying **your preferences**.

CHANCE

As we shall describe in details in each part of the experiment, some of the options you will be offered during the experiment return a payment that depends on **chance**. In particular, chance might determine the amount of cash that a given option pays, or the date on which this payment will be made.

For example, you might face an option that returns benefit A with probability 50%, and benefit B with probability 50%.

To determine this, we will use the **roll of a die**. In particular, after all three parts of the experiment are completed, at the very end of the experiment, one of the participants will be randomly selected to act as the **assistant**, and his/her task will be exactly to roll a die to determine the benefits returned by a given option.

To illustrate, consider the example above the option that returns benefit A with probability 50%, and benefit B with probability 50%. Then, the assistant will roll a 6-faced die, and if he/she obtains faces 1-2-3 you will receive benefit A; if instead he/she obtains faces 4-5-6, you will receive benefit B.

Depending on the questions, the probabilities involved could be different e.g., 5%, 30%, etc. but the outcome will always be determined by the roll of a die. We may use a 6-faced, 8-faced, 10-faced, or 12-faced dice depending on the question. You will be able to observe this process, and, if you wish, to inspect the dice used by the assistant.

PAYMENTS

Your payoff in the experiment will be determined as follows:

- After youve answered all the questions in all parts of the experiments, we will choose randomly, with equal probability, one question from all the questions youve answered. This will be done by the roll of a die by the assistant, as described above. You will then receive the benefit paid by the option that you have chosen for that question (which may depend on chance).

- Your total earning will consist of the amounts above plus a \$10 participation fee if you complete the experiment. This participation fee will paid to you at the end of the experiment **today** independently of the benefit received in the rest of the experiment. The benefit paid by the selected option could either be paid today, or be paid in the future, depending on the question.

PAYMENTS IN FUTURE DATES

Some of the options in the experiment involve payments to be made in **future** dates. For example, you might face an option that pays \$15 in 2 weeks. To receive this payment, you will be allowed to pick up the cash from **this** lab, at any point during office hours starting from the day specified onwards.

For example, if you receive the option that pays \$15 in 2 weeks, you can come and pick up the cash in this lab at any point during office hours, starting from 2 weeks from today.

You will receive an email to remind you of the approaching date. All possible payment dates will coincide with a day that the school is open.

To guarantee an accurate payment, we will save all the payment information until the date of the payment, but we will keep it **separately** from the rest of the data collected from the experiment, and it will be destroyed once all payments have been made.

You will also be provided with the contact details of Prof. **Daniel Gottlieb**, who is one of the persons conducting this research, and who will be responsible to ensure that you receive your payment. Please feel free to contact Prof. Gottlieb if any problem arises with your payment.

Instructions for Part 1

Instructions for Part I

There are 5 questions in this part. Please answer all of them. In each question you are asked to choose between two options by checkmarking your preferred one. If this question is selected for payment, the chosen option will pay a given amount of money on some date of the future.

The questions may look similar to this:

Question EXAMPLE

Payment: **$16.** Payment date:

Option A			Option B
13 days	☐	☐	75% chance of 20 days 25% chance of 10 days

Both options above will pay $16, as written above the question. Where they differ is on the date of the future payment.

Option A above will pay in 13 days. This means that if you choose this option, and this question is selected for payment, then you will receive $16 in 13 days.

Option B instead involves a payment date that will depend on chance: it above pays in 10 days with probability 25%, and in 20 days with probability 75%. If you choose this option, and this question is selected for payment, then chance will determine the payment date.

Recall that this payment date will be determined by the roll of a die done at the end of the experiment. At that point you will learn the payment date and the amount. This means that even if this date of payment is unknown before you answer, **at the end of the experiment you will learn exactly when the payment will be made.**

Instructions for Part 2

Instructions for Part II

There are 6 questions in this part. Each question is a list of **21 choices**, one in each row. For each decision row you will be asked to choose either Option A, on the left, or Option B, on the right. You make your decision by checking the box next to the option that you want. You may choose Option A for some decision rows and Option B for other rows.

In all questions, the 21 choices are presented as a list, in which Option A (on the left) **remains the same**, while Option B (on the right) **changes in each row**. For example, you might face the following question:

Row	Option A			Option B
1	$8.00 in 10 days	☐	☐	$8.00 in 20 days
2	$8.00 in 10 days	☐	☐	$8.25 in 20 days
3	$8.00 in 10 days	☐	☐	$8.50 in 20 days
4	$8.00 in 10 days	☐	☐	$8.75 in 20 days
5	$8.00 in 10 days	☐	☐	$9.00 in 20 days
6	$8.00 in 10 days	☐	☐	$9.25 in 20 days
7	$8.00 in 10 days	☐	☐	$9.50 in 20 days
8	$8.00 in 10 days	☐	☐	$9.75 in 20 days
9	$8.00 in 10 days	☐	☐	$10.00 in 20 days
10	$8.00 in 10 days	☐	☐	$10.25 in 20 days
11	$8.00 in 10 days	☐	☐	$10.50 in 20 days
12	$8.00 in 10 days	☐	☐	$10.75 in 20 days
13	$8.00 in 10 days	☐	☐	$11.00 in 20 days
14	$8.00 in 10 days	☐	☐	$11.25 in 20 days
15	$8.00 in 10 days	☐	☐	$11.50 in 20 days
16	$8.00 in 10 days	☐	☐	$11.75 in 20 days
17	$8.00 in 10 days	☐	☐	$12.00 in 20 days
18	$8.00 in 10 days	☐	☐	$12.25 in 20 days
19	$8.00 in 10 days	☐	☐	$12.50 in 20 days
20	$8.00 in 10 days	☐	☐	$12.75 in 20 days
21	$8.00 in 10 days	☐	☐	$13.00 in 20 days

As you can see, Option A, on the left is always the same, while Option B, on the right, changes: the amount of money it pays **increases** as you proceed down the rows. Your task is to choose **in each row** whether you prefer Option A or Option B.

Some of these questions, like the ones above, involve different amounts to be paid at different dates. These dates could be in the future, or could be marked as 'today: in this case, they would be paid at the end of the experiment today.

Other questions involve a fixed amount of money to be paid, but with a payment date that depends on **chance**. Consider for example the following question:
Payment: **$15.** Payment date:

Row	Option A			Option B		
1	In 28 days	☐	☐	0%	chance of	15 days
				100%	chance of	45 days
2	In 28 days	☐	☐	5%	chance of	15 days
				95%	chance of	45 days
3	In 28 days	☐	☐	10%	chance of	15 days
				90%	chance of	45 days
4	In 28 days	☐	☐	15%	chance of	15 days
				85%	chance of	45 days
5	In 28 days	☐	☐	20%	chance of	15 days
				80%	chance of	45 days
6	In 28 days	☐	☐	25%	chance of	15 days
				75%	chance of	45 days
7	In 28 days	☐	☐	30%	chance of	15 days
				70%	chance of	45 days
8	In 28 days	☐	☐	35%	chance of	15 days
				65%	chance of	45 days
9	In 28 days	☐	☐	40%	chance of	15 days
				60%	chance of	45 days
10	In 28 days	☐	☐	45%	chance of	15 days
				55%	chance of	45 days
11	In 28 days	☐	☐	50%	chance of	15 days
				50%	chance of	45 days
12	In 28 days	☐	☐	55%	chance of	15 days
				45%	chance of	45 days
13	In 28 days	☐	☐	60%	chance of	15 days
				40%	chance of	45 days
14	In 28 days	☐	☐	65%	chance of	15 days
				35%	chance of	45 days
15	In 28 days	☐	☐	70%	chance of	15 days
				30%	chance of	45 days
16	In 28 days	☐	☐	75%	chance of	15 days
				25%	chance of	45 days
17	In 28 days	☐	☐	80%	chance of	15 days
				20%	chance of	45 days
18	In 28 days	☐	☐	85%	chance of	15 days
				15%	chance of	45 days
19	In 28 days	☐	☐	90%	chance of	15 days
				10%	chance of	45 days
20	In 28 days	☐	☐	95%	chance of	15 days

In this question, all options available involve a payment of a fixed amount of money, \$15, as written on top. In the case of Option A, in all rows the payment will be made in 28 days. In the case of Option B the payment date will instead depend on chance: for example, Option B in row 17 involves a payment in 15 days with probability 80

Notice that as we proceed down the rows, Option B changes by **increasing the probability that t payment is made on the sooner date**.

If one of these questions is selected for payment at the end of the experiment, then one row will then be chosen randomly, with equal probability, using the roll of a die (made by the assistant). You will then receive the Option you have selected for that row. If the Option you have selected depends on chance, as in Options B in the example above, then again this will be resolved using the roll of a die made by the assistant (just like in the rest of the experiment).

Finally, recall that in this experiment **there are no right or wrong answers**. We are interested in studying **your preferences**.

C.1 Instructions for Part 3

Instructions for Part III

There are 6 questions in this part. As opposed to the previous parts of the experiment, in this part all questions involve options that, if selected for payment, will be paid out **today** at the end of the experiment. Like in Part II, each question is a list of **21 choices**, one in each row. Again, for each decision row you will be asked to choose either Option A, on the left, or Option B, on the right. As before, in all questions Option A remains the same, while Option B varies.

Consider for example the following question:

Row	Option A			Option B		
1	$9	☐	☐	0% 100%	chance of chance of	$15 $4
2	$9	☐	☐	5% 95%	chance of chance of	$15 $4
3	$9	☐	☐	10% 90%	chance of chance of	$15 $4
4	$9	☐	☐	15% 85%	chance of chance of	$15 $4
5	$9	☐	☐	20% 80%	chance of chance of	$15 $4
6	$9	☐	☐	25% 75%	chance of chance of	$15 $4
7	$9	☐	☐	30% 70%	chance of chance of	$15 $4
8	$9	☐	☐	35% 65%	chance of chance of	$15 $4
9	$9	☐	☐	40% 60%	chance of chance of	$15 $4
10	$9	☐	☐	45% 55%	chance of chance of	$15 $4
11	$9	☐	☐	50% 50%	chance of chance of	$15 $4
12	$9	☐	☐	55% 45%	chance of chance of	$15 $4
13	$9	☐	☐	60% 40%	chance of chance of	$15 $4
14	$9	☐	☐	65% 35%	chance of chance of	$15 $4
15	$9	☐	☐	70% 30%	chance of chance of	$15 $4
16	$9	☐	☐	75% 25%	chance of chance of	$15 $4
17	$9	☐	☐	80% 20%	chance of chance of	$15 $4
18	$9	☐	☐	85% 15%	chance of chance of	$15 $4
19	$9	☐	☐	90% 10%	chance of chance of	$15 $4
20	$9	☐	☐	95%	chance of	$15

As you can see, in this question Option A remains the same in all rows, at $9, while Option B varies: it pays an amount of dollars that depends on **chance**. It pays two different amounts, $4 and $15, with varying probabilities. In particular, as we proceed down the rows, **the probability of receiving the higher payment increases**. This means that, as we proceed down the rows, Option B pays the higher amount with a higher and higher probability. In particular, notice that in the **first** row Option B pays **with certainty $4**, while in the **last** row it pays **with certainty $15**. Your task is to choose **in each row** whether you prefer Option A or Option B.

Notice that in the example above, Option A does not depend on chance. However, there will be questions in this part in which also Option A depends on chance.

If one of these questions is selected for payment at the end of the experiment, then one row will be selected at random, with equal probability. This will be done again using the roll of a die (made by the assistant). You will then receive the option you have selected for that row. If this option involves chance, this will also be resolved using the roll of a die (made by the assistant). For example, if you choose 80% chance of $15 and 20% chance of $4 for row 17, and this question and this row are selected for payment, then with probability 80% you will receive $15, while with probability 20% you will receive $4. These payments will be made today at the end of the experiment.

Finally, recall that in this experiment **there are no right or wrong answers**. We are interested in studying **your preferences**.

References

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